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COSMIC RAY DIFFUSION - REPORT OF THE WORKSHOP IN COSMIC RAY DIFFUSION THEORY

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16. Abstract

A workshop in Cosmic Ray Diffusion Theory was held at Goddard Space Flight Center on May 16-17, 1974. Topics discussed were: 1) Cosmic ray measurements as related to diffusion theory; 2) quasi-linear theory, nonlinear theory, and computer simulation of cosmic ray pitch-angle diffusion; and 3) magnetic field fluctuation measurements as related to diffusion theory. This report summarizes the proceedings.

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PROLOGUE

The highlights of the Workshop in Cosmic Ray Diffusion Theory as reported here were prepared from tape recordings and our personal notes of the proceedings, together with preprint material generously supplied by the participants. We have tried to be objective reporters and to weight the material with an emphasis commensurate with that it received at the Workshop. Despite our best efforts, omissions, misinterpretations, and subjective personal impressions have undoubtedly crept into this account. We apologize for such failings.

Many persons contributed to the success of the Workshop, and foremost among these we thank the participants, without whom there could have been no Workshop. We also especially thank Dr. L. A. Fisk and Dr. J. W. Belcher for their efforts in preparing the very illuminating review talks. Our colleague, Dr. Thomas Kaiser, assisted on numerous occasions with advice and criticism. We thank Dr. George F. Pieper, Director of Sciences, GSFC, and Mrs. Evelyn Peters for their gracious hospitality. We profited throughout from the support and encouragement of Dr. Aaron Temkin, Acting Chief, Theoretical Studies Group. To Mrs. Sandra Walter, who served as Workshop secretary, we extend a particular word of thanks.

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COSMIC RAY DIFFUSION—REPORT OF THE WORKSHOP IN COSMIC RAY DIFFUSION THEORY

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INTRODUCTION

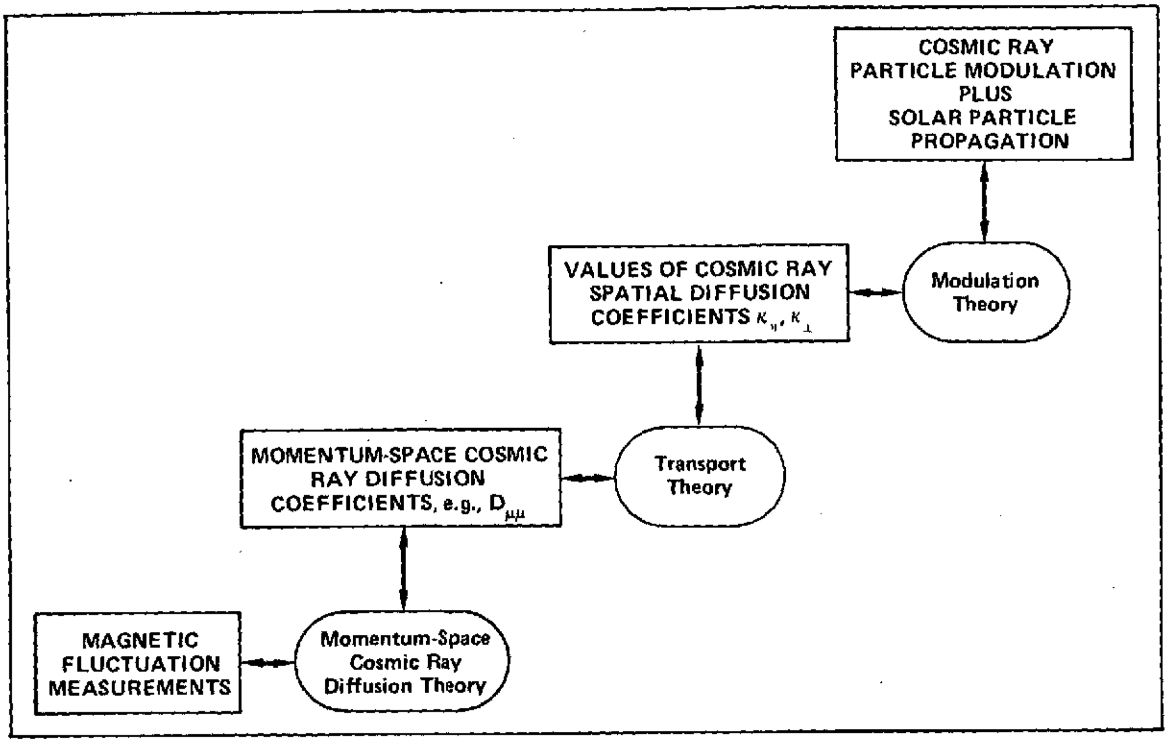
A Workshop in Cosmic Ray Diffusion Theory was held at the Goddard Space Flight Center on May 16-17, 1974. The primary purpose in organizing this Workshop was to gather together individuals working in the field for informal presentation, discussion, and clarification of recent developments, particularly in nonlinear theory. The nineteen attendees are all actively pursuing research in cosmic ray diffusion theory or in closely attendant areas.

As plans for the meeting progressed, it was decided to expand its scope and include a review of the pertinent experimental evidence. Dr. L. A. Fisk was invited to present a summary of particle observations relevant to cosmic ray diffusion theory and Dr. J. W. Belcher was asked to discuss magnetic measurements germane to the area. With the exception of these two presentations, the Workshop operated in a totally unstructured manner. Dialogue was encouraged in the hope that differences of opinion, which have been highly visible in the past, might thus be resolved.

As this account will attest, there was a considerable consensus as to where the field stands and where it should go. Divergences of views still exist as in any new and active research area, but even here progress was made in resolution.

There was a strong consensus that the area of cosmic ray diffusion theory is an important one: diffusion theory is a connecting link between two other active, major areas of interplanetary research, cosmic ray particle modulation and magnetic fluctuation measurements. Also important is the fact that the problems encountered in cosmic ray diffusion theory are common to plasma turbulence theory of interest in fusion research. Some of the mathematical techniques used in treating plasma turbulence have been adapted to the cosmic ray problem, but it now seems possible that methods developed independently for the cosmic ray problem are of value and relevance to plasma turbulence.

The following pages summarize the major deliberations of the Workshop. Since the topic of cosmic ray diffusion theory is a rather narrow one and the participants were generally familiar with background material, discussion tended to be quite specialized and a bit fragmented. While this writeup is intended as a summary, it must necessarily reflect to some extent this specialized character of the proceedings.



The accompanying diagram is intended to help orient those readers not active in the field. One may proceed logically from either bottom left to top right or vice versa, depending upon one's prejudices as to whether the field or particle observations are more fundamental. We shall begin at the bottom left, although the rough format of the Workshop began at the top right. Quantities in rectangular boxes are to be regarded as "products"–either input, output, or intermediate. Quantities in ovals are "processes"–theoretical connections between pairs of "products."

The highly irregular time fluctuations in the interplanetary magnetic field measured by spacecraft are generally regarded as random spatial fluctuations, approximately stationary in time, being convected out by the super-Alfvenic solar wind flow. Some statistical properties of these fluctuations have been measured.

Cosmic rays see the irregularities as scattering centers, which, in the solar wind frame, alter their momenta in an energy preserving way. If the fluctuations are small and not strongly correlated, it is plausible that a diffusion theory should describe statistically these deflections in momentum. The major concern of most participants has been the proper relation of the momentum-space diffusion coefficients to the statistical properties of the magnetic fluctuations.

For much of the phase space of any given cosmic ray, quasi-linear theory would seem correct. This theory is a bootstrap procedure in which the Lorentz force on a particle due to

the fluctuating magnetic field is evaluated according to (q/c) $\mathbf{v}_0$ $(\tau) \times \delta\mathbf{B}$ $[\mathbf{r}_0$ $(\tau)]$, where $\mathbf{v}_0$ (τ), $\mathbf{r}_0$ (τ) is the particle trajectory neglecting the effects of the fluctuations. When the interaction is weak, that is, the cosmic ray moves out of a region of statistically correlated $\delta\mathbf{B}$ before the fluctuating field largely affects its orbit, the particle trajectory does not differ significantly from $\mathbf{r}_0(\tau)$, $\mathbf{v}_0(\tau)$ and the approximation is a valid one. However, there are regions of phase space for this problem, such as 90° pitch angle with respect to the average background magnetic field (assumed spatially uniform), where the duration of an interaction is arbitrarily long in the quasi-linear approximation, and the theory is therefore generally thought to be invalid. It is for these regions of phase space that nonlinear theories have been proposed.

The next step, after momentum-space diffusion coefficients have been derived, is to calculate spatial diffusion coefficients $\kappa_\parallel, \kappa_\perp$ by coarse graining the momentum-space description over directions of $\mathbf{p}$. The only comment on this phase of the theory at the Workshop was Earl's statement referred to in the concluding section of this report.

Finally, one introduces the spatial diffusion coefficients into the modulation equation. This equation incorporates convection and adiabatic energy changes as well as spatial diffusion of the particles. There was no critical discussion of the modulation equation.

Solutions of the modulation equation can be converted to particle fluxes for comparison with cosmic ray observations. A coherent picture is one in which the magnetic observations lead through the various diagramed stages to solutions of the modulation equation in agreement with particle observations.

Three areas of this block diagram were treated extensively at the Workshop. In the order of their discussion they are: 1) particle observations; 2) momentum-space diffusion theory including quasi-linear theory, nonlinear theory, and numerical simulation; and 3) magnetic observations.

PARTICLE OBSERVATIONS IN RELATION TO DIFFUSION THEORY

Fisk summarized the cosmic ray evidence for particle modulation by magnetic irregularities. He pointed out that the data lead directly only to values for the spatial diffusion coefficients $\kappa_\parallel$ and $\kappa_\perp$ parallel and perpendicular, respectively, to the average magnetic field. The $\kappa_\parallel$ and $\kappa_\perp$ are in turn integral functions of the momentum-space diffusion coefficients: $\kappa_\parallel$ for example, is an integral function of the pitch-angle diffusion coefficient $D_{\mu\mu}$, one quantity under scrutiny in the present Workshop. No fine-grained information about the pitch-angle dependence of $D_{\mu\mu}$ is therefore available from the cosmic ray data as it now exists. Fisk also cautioned about hypothesizing spatial diffusion in cosmic ray phenomena where the basic assumptions of spatial diffusion theory are invalid, specifically those in which the observer is within one mean free path λ_{mfp} of injection boundaries. He cited low energy (<10-20 MeV/nucleon) solar cosmic ray events as generally suspect in this regard.

One appears to be on safest ground in applying modulation theory to galactic cosmic rays. Several different types of observations—total intensity reduction between the interstellar medium and the orbit of earth, time variations of the flux over the solar cycle, and measurements of the radial flux gradient as well as its radial anisotropy—support the idea that modulation is occurring and lead to estimates of κ. Recent Pioneer-10 measurements of the gradient in the integral proton flux $j > 60$ MeV (Teegarden et al., 1973) lead to a value of κ of 3×10^{22} cm^2/s. This value should be characteristic of the 1 GeV average energy of particles measured. However, the value is five times larger than the value of κ calculated by using the power spectrum of magnetic fluctuations obtained by Jokipii and Coleman (1968) from the 1964 Mariner-4 observations in Jokipii's (1966) theoretical formula. To produce agreement, there should be less pitch-angle scattering than is predicted by simple theory. Theoretical extrapolation of the 3×10^{22} cm^2/s figure to lower energies leads to a value of $\kappa \cong 7 \times 10^{20}$ cm^2/s in the 50 MeV/nucleon region, consistent with Pioneer-10 measurements of the differential helium flux gradient (McDonald et al., 1974). (Such an extrapolation is model dependent.)

Those solar flare events to which the diffusion approximation is applicable seem consistent with the above values of κ. The three solar events observed by Lupton and Stone (1973) yield a value of $\kappa \cong 5 \times 10^{20}$ cm^2/s, apparently rigidity independent in the low-energy (5 MeV/nucleon) range.

A consistent picture based on modulation theory emerges. Even the anomalously high fluxes of oxygen and nitrogen around 10 MeV/nucleon fit in if one assumes that these species are singly ionized. According to a recent theory of Fisk, Kozlovsky, and Ramaty (1974) such an ionization state is possible: neutral oxygen and nitrogen enter the heliosphere, are singly ionized, energized, and swept out before additional ionization occurs, and then re-enter as high-rigidity cosmic rays.

The cosmic ray modulation varies with time over the solar cycle. In contrast, the solar wind bulk speed and the overall magnetic fluctuation level remain virtually constant. Measurement of these two quantities has been restricted to the near-Earth region, however, while Pioneer-10 observations indicate that the bulk of the particle modulation occurs beyond the orbit of Jupiter.

According to Fisk, the major discrepancy is with the magnetic observations: the mean free path ($\lambda_{mfp} \equiv 3\kappa/v$) of 0.1 AU that one typically determines from the particle observations is roughly an order of magnitude larger than that obtained using the magnetic power spectrum. There are two obvious ways to produce agreement: either increase λ_{mfp} as obtained from the power spectrum or reduce λ_{mfp} as deduced from the particles. There was considerable later discussion about λ_{mfp} as calculated from the magnetic observations. There seemed to be a consensus that $D_{\mu\mu}$, and hence λ_{mfp}, are extremely sensitive to the turbulence model invoked and that the slab model used by Jokipii and Coleman (in which the magnetic fluctuations are plane waves, linearly polarized transverse to the average magnetic field) probably gives the largest value of $D_{\mu\mu}$ and hence the smallest values of κ and λ_{mfp}.

There was no one who argued that λ_{mfp} as deduced from the particle observations is less than 0.1 AU. To the contrary, Roelof cited solar cosmic ray events in which large (factor of 2), short term (1-3 hours), anisotropic increases in flux occurred superposed on the decay phase of a longer event. Since there were no accompanying anomalies in the plasma or magnetic field data which might suggest a local source, Roelof has interpreted this phenomenon, which occurs at both low (1 MeV) and high (1 GeV) energy, as scatter-free propagation of cosmic rays impulsively injected at the sun. This interpretation leads to a value of at least 1 AU for λ_{mfp}.

Roelof also asserted that backscatter measurements (McCracken et al., 1967; Quenby et al., 1974)–observations of solar cosmic rays scattered from regions of the interplanetary medium beyond the orbit of Earth–suggest a long λ_{mfp} of typically 3 AU.

There seems to be an inexplicable inconsistency between the cases cited by Roelof and those referred to by Fisk. A suggestion was made that κ might depend on the pitch-angle distribution of the particles, being large for beams well collimated along the average **B**-field. The question also arose as to what one means by a mean free path in the cases referred to by Roelof. It was suggested that in the backscatter experiments particles have traveled several λ_{mfp} in traveling from source to receiver. It was generally thought that while these two possibilities might explain factor of 2 differences in λ_{mfp}, they were inadequate for the factor of 10 discrepancies that separate the observations and interpretations cited by Fisk and Roelof.

With regard to the difference between the values of λ_{mfp} determined via the particles and the magnetic power spectra, the question arose as to whether the particle and magnetic measurements incorporated consistent statistical averages and whether either or both was equivalent to the ensemble averaging assumed in deriving the theory. The consensus seemed to be that this was a fundamental point, one thus far unexplored in cosmic ray physics.

Owens remarked that he has examined fluctuations (scintillations) in the cosmic ray counting rate (Owens and Jokipii, 1972; Owens, 1974). These fluctuations, which are larger than those expected from simple counting statistics, are essentially fluctuations in the cosmic ray distribution function. They play a central role in the theory of cosmic ray propagation in random fields; therefore, their power spectrum can be related to the power spectrum of magnetic fluctuations in a straightforward manner. This theoretical relationship appears to be borne out by observations, a result that lends credence to the basic theory.

QUASI-LINEAR THEORY OF PITCH-ANGLE DIFFUSION

An attempt was made to delineate the regime of validity, if any, of the quasi-linear theory of pitch-angle diffusion (see remarks on quasi-linear theory on pp. 2-3). It was the consensus that a quasi-linear diffusion theory is applicable provided that one is not considering 1) cosmic rays whose pitch angles are too near 90° with respect to the average background field or 2) cosmic rays of too low an energy.

There was unanimity on the first restriction. Jones spoke of a calculation that he had performed with Birmingham: the next correction to quasi-linear theory was calculated and shown to be asymptotically small provided that τ_d, the characteristic time for a particle's trajectory to deviate from its orbit in the average field, is large compared to τ_c, the coherence time between a cosmic ray particle and the fluctuations. For the pitch-angle diffusion case, the following pitch-angle restriction results:

$$\mu = \cos\lambda \gtrsim \frac{\langle \delta B^2 \rangle^{1/2}}{\langle B \rangle}$$

where λ is the pitch angle, $\langle \delta B^2 \rangle^{1/2}$ is the rms fluctuating magnetic field amplitude, and $\langle B \rangle$ the average field strength.

There was much less agreement on the second restriction and no one was able to quantify it. As particle kinetic energy $T \to 0$, $\tau_c \to \infty$ for particles interacting with static magnetic fluctuations. But $\tau_d \to \infty$ also in this energy limit because of the vanishing of the Lorentz force. The validity of quasi-linear theory, therefore, rests on the behavior of the ratio τ_c/τ_d. While the value of this ratio depends on the precise nature of the magnetic turbulence considered, it was the general feeling that $\tau_c/\tau_d \to \infty$ for situations of interest, in which case quasi-linear theory is invalid at low energies.

Owens pointed out an indication from the data that quasi-linear theory cannot be completely correct: the measured spectrum of cosmic ray fluctuations is well behaved in the vicinity of the gyrofrequency, where an infinity should occur according to quasi-linear theory.

Klimas spoke of a result obtained by Goldstein, Sandri, and himself from the quasi-linear diffusion theory of cosmic rays with 90° pitch angles interacting with a general spectrum of static, spatially homogeneous, magnetic turbulence. This group acknowledges the difficulty of quasi-linear theory near $\lambda = 90°$. They have continued to explore this parameter regime, however, with the attitude that the quasi-linear results should manifest the gross physics and thus act as a guideline for nonlinear theories. They find that $D_{\mu\mu}$ always has a δ-function singularity at $\mu = 0$ ($\lambda = 90°$), provided that the turbulence has a component along the background field such that particle mirroring can occur.

Along similar lines, Lee discussed work that he has recently performed with Völk. Two results are of note. The first is that an isotropic MHD turbulence model is probably inappropriate for the interplanetary medium. The reason is that isotropy requires that both Alfven and magnetosonic waves be present in equal intensities. The observations are that the interplanetary MHD turbulence is largely Alfvenic.

The second result is from a quasi-linear calculation of $D_{\mu\mu}$ which takes into account the propagating ($\omega \neq 0$) nature of the Alfven and magnetosonic modes. The δ-function of Klimas et al. is identified as a resonant contribution due to the magnetosonic modes. Because $\omega \neq 0$, the δ-function turns into a resonance of finite width and amplitude

displaced slightly from $\mu = 0$. Quasi-linear theory remains in difficulty, for there are regions near $\mu = 0$ where $D_{\mu\mu}$ is rigorously zero if only the Landau (n = 0) resonance is considered. Inclusion of higher order resonances presumably fills in such regions slightly but not to the extent that cosmic rays can efficiently diffuse through them.

A somewhat dissenting stance was taken by Roelof, who argued that so long as one did not insist on a Markovian, diffusion description of pitch-angle scattering, quasi-linear theory might be valid in all pitch-angle ranges. He cited his past work (Roelof, 1966, 1968) in which the quasi-linear approximation $\delta f \ll \langle f \rangle$ was made but the adiabatic approximation $\partial \ln \langle f \rangle / \partial t \ll 1/\tau_c$ was not. An integro-differential equation for $\langle f \rangle$ results. The solution of this equation is a "reasonable" appearing $\langle f \rangle$. There was objection that the quasi-linear approximation breaks down over the long time intervals that it takes $\langle f \rangle$ to relax when μ becomes small. Even though this failure occurs, the $\langle f \rangle$ equation may yield "reasonable" (bounded) results. Boundedness of $\langle f \rangle$ is a necessary constraint, but not equivalent to correctness.

NONLINEAR THEORIES OF PITCH-ANGLE DIFFUSION

Several attempts to correct the inadequacies of quasi-linear diffusion theory were discussed. The newest of these was a recent derivation by Jokipii. He considered the slab model and argued that the difficulties of quasi-linear theory in the vicinity of 90° pitch angles were obviated if one evaluated the pitch-angle diffusion coefficient according to the prescription $D^J_{\mu\mu} = \langle (\Delta\mu)^2 \rangle / 2\Delta t$, with μ taken to be the cosine of the angle between **p** and the *total* magnetic field, average plus fluctuating. The procedure is to be contrasted with the usual one of evaluating $D_{\mu\mu} = \langle (\Delta\mu_A)^2 \rangle / 2\Delta t$, where μ_A is the cosine of the angle between **p** and the *average* field only. While $D_{\mu\mu}$ oscillates wildly with time for $\mu_A \to 0$, a manifestation of the particle's attempt to maintain constant μ, $D^J_{\mu\mu} \to 0$ in a few cyclotron periods. To get diffusion at $\mu = 0$, one must go to the higher order $(\delta \mathbf{B})^4$ in this theory. This step has not been taken as yet.

Jones commented that in the critical region near 90° pitch angles, the relative difference $(\mu - \mu_A)/\mu_A$ can be arbitrarily large. This difference is important because in a transport equation the independent variable should be defined with respect to a fixed coordinate system; μ_A satisfies this criterion, while μ does not.

Three other nonlinear theories were discussed by Völk, Owens, and Jones. In each theory, $D_{\mu\mu}$ is proportional to

$$\int_0^\infty d\tau \, \langle \delta \mathbf{B}(x) \, U(t, \tau) \, \delta \mathbf{B}(x) \rangle .$$

The quantity in brackets is an ensemble average: the autocorrelation of the magnetic field evaluated according to a prescription which depends on U. The operator U differs among the three theories, but in each it depends in some way upon the fluctuations themselves.

This is in contrast with quasi-linear theory, where U is independent of the fluctuations and simply propagates $\delta\mathbf{B}$ (x) backward along the orbits of particles in the average background magnetic field. All three theories arrive at a Markovian, diffusion description of pitch-angle scattering, but the strength of the diffusion in the 90° pitch-angle region varies significantly among them. At small pitch angles, they reduce, as they should, to quasi-linear theory.

Völk's (1973) approach is adopted from a theory of strong plasma turbulence proposed by Dupree (1966) and later refined by Weinstock (1969). In this theory, U propagates $\delta\mathbf{B}$ (x) along a statistically scattered set of orbits. At the Workshop, Völk illustrated his theory for the case of slab model Alfven-wave turbulence, for which $D_{\mu\mu}$ ($\mu = 0$) = 0 according to quasi-linear theory. What the nonlinear formalism does is to fill in the void around $\mu = 0$ in the $D_{\mu\mu}$ versus μ curve. Völk expressed some misgivings about the applicability of the Dupree method, which was originally devised to treat narrowband electrostatic turbulence, to the case of wideband magnetic fluctuations. He expressed more confidence in a simple heuristic procedure which fills in the region $-\mu^* \leqslant \mu \leqslant \mu^* = \langle\delta\mathbf{B}^2\rangle^{1/2}/\sqrt{2}\,\langle B\rangle$ with the constant value $D^{QL}_{\mu\mu}$ ($\mu = \mu^*$). The physical motivation for this fill-in method is that each large-pitch-angle cosmic ray is scattered by the fluctuations in such a way that it samples all values $-\mu^* \leqslant \mu \leqslant \mu^*$ rather uniformly.

In Owens' (1974) formulation U is assumed to propagate $\delta\mathbf{B}$ (x) along an orbit in the average background field (just as in quasi-linear theory), but at the same time it damps the correlation by the factor exp ($-\alpha_0 \tau$). This assumed damping is due to the scattering of the cosmic rays by the fluctuations. The damping decrement α_0 has initially an unspecified dependence on the properties of the magnetic turbulence. The value of κ is determined canonically and depends, of course, on α_0. The value of α_0 is determined in an a posteriori but self-consistent manner by invoking the additional, independent relationship

$$\kappa = \frac{1}{3}\lambda v = \frac{1}{3}\frac{v^2}{\alpha_0}$$

Owens has applied this methodology to slab model magnetic turbulence. For particles of rigidity > 0.1 GV in the interplanetary medium, $\kappa_\perp$ is changed but slightly from the value determined from quasi-linear theory. The modification to $\kappa_\parallel$ depends critically on the spectrum of magnetic turbulence. For power law spectra that decay more slowly than $1/k^2$, the correction to the quasi-linear result is small for all cosmic ray energies. For more rapidly decaying spectra, quasi-linear theory leads to an infinite value of $\kappa_\parallel$ (owing to the nonintegrable singularity in $D_{\mu\mu}$ at $\mu = 0$), while the nonlinear theory predicts a finite value. In the context of nonlinear theory, scatter-free propagation ($\kappa_\parallel \to \infty$) therefore does not occur.

Lerche criticized Owens' approximation procedure on two mathematical counts: 1) it fails to preserve consistently the proper ordering in the small expansion parameter and 2) it leads to the conclusion that

$$\int \langle \delta n_1^2 \rangle \, d^3p = 0$$

that is, the mean-square density fluctuation integrated over all cosmic ray momenta is zero.

Jones spoke of the nonlinear procedure developed by Kaiser, Birmingham, and himself (Jones et al., 1973; Kaiser, 1973). In this approximation method, U is assumed to propagate $\delta\mathbf{B}$ (x) back along the orbit $\mathbf{x}^*$ (t, τ) in the partially averaged field $\mathbf{B}_p$. The partially averaged field is the magnetic field which results when an averaging is performed over all members of the ensemble with the same value $\delta\mathbf{B}$ (x) at the field point x. If the statistics of the turbulence are Gaussian and spatially homogeneous, $\mathbf{B}_p$ $(x', x) = \langle B \rangle + \delta B(x) \cdot \mathbf{C}$ $(x' - x)$, where $\mathbf{C}$ $(x' - x)$ is the normalized correlation tensor for the fluctuating magnetic field. The orbit $\mathbf{x}^*$ is then obtained by solving Newton's equation $\ddot{\mathbf{x}}^* = (q/\gamma mc)\, \dot{\mathbf{x}}^* \times \mathbf{B}_p\, (\mathbf{x}^*, \mathbf{x})$. After this procedure is carried out for each value of $\delta\mathbf{B}$ (x), a final averaging over the statistical distribution of the $\delta\mathbf{B}$ (x)'s is performed to complete the ensemble averaging process.

For slab model turbulence, Newton's equation for a particle in the partially averaged field can be handled semi-analytically. In the general case of three-dimensional turbulence, however, the equation is so complicated that recourse to numerical integration is necessary.

Results of calculations in a three-dimensional, isotropically turbulent field with a power spectral index of -2 were presented by Kaiser for a wide range of rigidities and random field strengths. The most significant feature of these results was the presence of a pronounced peak in $D_{\mu\mu}$ (μ) at $\mu = 0$. This peak is characterized by a rigidity-independent width equal to and a height roughly proportional to the relative random field strength, $\langle \delta B^2 \rangle^{1/2} / \langle B \rangle$. In the low-rigidity limit, the rigidity dependence of the peak height is linear.

COMPUTER SIMULATION EXPERIMENTS

Kaiser described a series of computer simulation experiments in which the pitch-angle diffusion of charged particles in a random magnetic field was investigated.

The basic approach in these experiments is to generate a statistical ensemble of random magnetic field realizations in which the power spectrum of the fluctuations has a prescribed form and the strengths of the average and rms random components of the field take on predetermined values. Orbits of charged particles diffusing in the random fields are followed, and from their evolution in time a pitch-angle diffusion coefficient is derived.

In the simulations reported a slab model magnetic field was used, with the random component linearly polarized and the power spectrum of the fluctuations given by

$$P(k) = \frac{2L_c \langle \delta B^2 \rangle}{1 + k^2 L_c^2}.$$

The results presented for a variety of rigidities and random field strengths contradict the predictions of quasi-linear theory in the region $|\mu| \lesssim \langle \delta B^2 \rangle^{1/2} / \langle B \rangle$. The nonlinear theory of Jones, Kaiser, and Birmingham, however, was shown to agree quite well with the simulation data for all values of μ.

Values of $D_{\mu\mu}$ ($\mu = 0$) from the experiments were presented for the following paired values of $\eta = \langle \delta B^2 \rangle^{1/2} / \langle B \rangle$, $\epsilon = L_c \cdot q\langle B \rangle / pc$: $\eta = 0.10, \epsilon = 1.0$; $\eta = 0.15, \epsilon = 2.0$; $\eta = 0.30, \epsilon = 1.0$. The results (in properly scaled units) were $D_{\mu\mu} = (8.7 \pm 1.2) \times 10^{-4}$, $(3.6 \pm 0.7) \times 10^{-3}$, and $(2.2 \pm 0.3) \times 10^{-2}$ respectively. For the same η, ϵ pairs the theory of Owens gives $D_{\mu\mu} = 5.0 \times 10^{-5}$, 5.1×10^{-4}, and 4.0×10^{-3}. The theory of Völk yields $D_{\mu\mu} = 1.2 \times 10^{-3}$, 3.0×10^{-3}, and 9.7×10^{-2}. The theory of Jones et al. gives $D_{\mu\mu} = 8.0 \times 10^{-4}$, 2.7×10^{-3}, and 2.2×10^{-2}.

Kaiser said that future simulations are in preparation in which the random field will be generalized beyond the slab model in order to study effects present only in fluctuating fields with longitudinal as well as transverse components.

SPECTRA OF FLUCTUATIONS IN THE INTERPLANETARY MAGNETIC FIELD

Belcher began by noting that the typical interplanetary-magnetic-field power spectrum in one dimension (dependent only on the radial component of the wave vector **k**) is relatively flat out to a value $\nu \cong 3 \times 10^{-5}$ Hz and falls off as $\nu^{-\alpha}$ (with α between 1.5 and 2.0) at higher frequencies. The low-frequency power ($< 10^{-6}$ Hz) is due primarily to large scale stream and sector structures which corotate with the sun. The physical origin of these structures is apparently well understood.

Of importance for the pitch-angle diffusion of cosmic rays in the interplanetary medium is the portion of the power spectrum $> 10^{-6}$ Hz. This part of the spectrum is dominated by the contribution of discontinuities and waves of scale size less than 10^{12} cm, which propagate internally to the main bodies of the streams. Nearly all of Belcher's presentation was devoted to these discontinuities and waves.

MHD discontinuities are of five types: tangential discontinuities, contact discontinuities, perpendicular shocks, inclined shocks, and rotational discontinuities. Of these, only tangential and rotational discontinuities are commonly present in the interplanetary medium. Cosmic rays of energy $\geqslant 4$ GeV/nucleon have gyroradii greater than the 10^{11} cm mean separation between tangential discontinuities. Only such energetic particles are pitch-angle scattered by tangential discontinuities. Rotational discontinuities and waves, on the other hand, interact with cosmic rays of all energies. It is thus important to determine the separate spectral contribution of each type of structure.

Belcher presented criteria for distinguishing between rotational and tangential discontinuities. He then reported statistical results of applying these criteria to 8713 changes observed in Mariner-5 data and 24,178 changes seen in the Pioneer-6 data. A significant conclusion of this study is that large events tend to be rotational rather than tangential. Large events

are those in which changes in the kinetic- and/or magnetic-energy densities across the discontinuity are larger than the energy density in the background field. The frequency of large events in this analysis was ~1 per hour.

On the other hand, a study of 200 directional discontinuities carried out by Burlaga (1971) was mentioned. These directional discontinuities differ from the changes referred to in the preceding paragraph in that relatively smooth conditions always existed on both sides of the events in Burlaga's study. Burlaga concluded that less than 25 percent of the directional discontinuities in his study are rotational.

A study by Fisk and Sari (1973) of the contribution of discontinuities to the *one-dimensional* magnetic field power spectrum concluded that discontinuities dominate the power only at frequencies $<10^{-4}$ Hz. At higher frequencies they are responsible for perhaps one half the observed power. At the low frequencies, the Larmor radii of particles are so large that they interact even with tangential discontinuities and hence all of the observed power is effective in pitch-angle diffusion. At the higher frequencies, resonant energies are lower, so tangential discontinuities are ineffective in scattering. But discontinuities are also non-dominant in this frequency regime. The result, as Fisk pointed out in discussion, is that subtracting tangential discontinuities from the one-dimensional spectrum lowers the pitch-angle diffusion coefficient a small amount, but not by the order of magnitude needed to bring the calculated κ into line with that obtained from the particle observations.

Belcher also reported on studies of the occurrence and properties of Alfven waves in the interval 5×10^{-5} Hz $< f < 1$ Hz. The magnetic microstructure is dominated by such Alfven waves when the measurements are made in high-velocity solar wind streams and on their trailing edges. In low-velocity regions of the interplanetary medium Alfven waves are also found, but they here have smaller amplitudes and are usually intermixed with other MHD structures.

Solodyna and Belcher (1974) have studied five 14.3-hour periods of magnetic data which were highly Alfvenic. The directional discontinuities encountered in these intervals were dominantly rotational so that 90 percent or more of the magnetic power was in the form of Alfven waves and rotational discontinuities. The **k** vectors of the Alfven waves had a statistical distribution in direction which was peaked in the equatorial plane along an axis intermediate between the radial direction and the direction $\pm\langle\mathbf{B}\rangle$ of the average magnetic field. (The sign is chosen so that the vector always has an outward radial projection.) This distribution is in accord with neither geometrical optics, which predicts the peak along the radial, nor with the picture of slab model turbulence, which would have the **k** vectors predominantly along $\langle\mathbf{B}\rangle$. The energy in Alfvenic fluctuations was found to decrease approximately as r^{-3}, a result consistent with undamped Alfven waves of solar origin propagating in a spherically symmetric solar wind.

Belcher concluded by acknowledging a need for more complete statistical studies of waves and discontinuities in the solar wind. The samples of data that have been studied are small and cover limited periods of time. Based on such limited data, Belcher's picture of

the solar wind is one that he termed "clumpy"–mostly rotational and Alfvenic in high-velocity streams, a mixture including tangential discontinuities in low-velocity streams.

Several participants stressed the need for further information about the tensor nature of the magnetic field fluctuations. Formulae for the diffusion coefficient involve the correlation tensor $\mathbf{P}(\mathbf{k})$ in a way which is sensitive to the vector $\mathbf{k}$ dependence. Unfortunately, little is known observationally about this dependence; and from single satellite observations little can be learned without recourse to gross assumptions.

Morfill urged that a systematic study of the polarization properties of the waves as a function of the angle between $\pm\langle\mathbf{B}\rangle$ and the radial should be carried out. Such a study would provide some further answer to the question of whether waves are behaving according to geometrical optics.

Sari discussed two results he has recently obtained. A cross correlation was found between the flux of 60-80 MeV protons measured on an IMP spacecraft close to the Earth, with values of $v_{sw}/\kappa_{\parallel}$ (v_{sw} is the solar wind speed) calculated using magnetic power spectra obtained from Pioneer observations. At the time of these observations, Pioneer was at 0.8 AU and the Earth-Sun-Pioneer angle was 15° W. The cross correlation peaked at a delay time of 1-2 days between the observations. The magnitude and sense of this delay can be explained in terms of the rotation of a nearly radial field line initially intersecting IMP to a later intersection with Pioneer. The fact that there is a correlation is indicative that modulation is occurring. The fact that the correlation is relatively sharp means that cosmic rays have not undergone much cross field diffusion in the time interval that it takes the magnetic field line to corotate from one spacecraft to the other; hence $\kappa_{\perp} < 5 \times 10^{19}$ cm^2/s.

Sari has also obtained values of the power spectrum ratio $P_{zz}/(P_{xx} + P_{yy})$ as a function of the angle θ between the radial direction and the direction z of the average interplanetary magnetic field. He finds that as $\theta \rightarrow 0$, $P_{zz} \rightarrow 0$; that is, the power is totally in the transverse x and y components of $\mathbf{B}$.

There was considerable inconclusive discussion of how the interplanetary magnetic field could remain in the radial direction for long periods of time. To many individuals, such a radial field seemed to contradict the idea of interplanetary field lines being rooted in the sun and corotating with it.

CONCLUSIONS

Discussion in the final session of the Workshop was aimed at tying together the opinions of all participants regarding both the present status of diffusion theory and the most important and promising directions for future study. There was some success along these lines, although considerable divergence of opinion exists on many issues.

There was a strong consensus that the present course of evaluating the spatial diffusion coefficients by relating them to the momentum-space diffusion coefficients is indeed the proper one. The fact that the value of κ so obtained is at least an order of magnitude too small to agree with the particle observations should not be regarded as catastrophic. It is now believed that κ is very sensitive to the assumptions made in the theory, and that in the above comparison the smallest possible theoretical value was obtained.

There was some discussion as to the proper relationship between κ and the momentum-space diffusion coefficients, once the latter are obtained. Earl claimed to have settled the issue of relating $\kappa_{\parallel}$ uniquely to $D_{\mu\mu}$. Since most participants seemed unfamiliar with this recent work (Earl, 1974), little consensus pro or con was obtained.

It was generally agreed that quasi-linear theory is a valid procedure for calculating velocity-space diffusion coefficients except for particles of large pitch angle ($>\cos^{-1} \langle \delta B^2 \rangle^{1/2} / \langle B \rangle$) or low kinetic energy. The low-energy cutoff has not been explored and most participants felt that this situation should be rectified. However, they also felt that this cutoff lies very low on the scale of cosmic ray energies.

In the region of large pitch angles, a nonlinear theory is necessary in order to remove singularities arising when quasi-linear theory is applied to most models. It seemed the consensus that this nonlinear theory would result in a stochastic, diffusion description of the particles, although Roelof's position was that such a Markovian description is not always appropriate.

Computer simulations are a norm against which the correctness of nonlinear theories can be compared. A strong recommendation that this phase of research receive increased emphasis was made. Since three-dimensional simulations require large amounts of computer usage, simulation can be a costly undertaking. The group felt that such expenditure is justified.

The matter of ensemble averages, which occur in both quasi-linear and nonlinear theories, and their relationship to measurements was discussed. Discussion indicated that the ensemble which the theorist invokes should perhaps be measurement specific; for example, the ensemble used in treating solar cosmic rays should be different from the one used for galactic cosmic rays. There seemed to be a total respect for this profound question but almost a total absence of useful procedures for exploring it. The attitude for the time being seemed to be to assume that measurement procedures are ergodic.

Success in coming up with a value of κ which agrees with the particle data seems to hinge above all on obtaining a simple, realistic model of the magnetic turbulence in the interplanetary medium. There seems to be very little experimental information about the dependence of the magnetic field correlation tensor $\mathbf{P}(\mathbf{k})$ on the vector properties of $\mathbf{k}$. As was stated many times over the two days of the Workshop, the simple assumption that the $\mathbf{k}$ vectors are aligned totally along the average magnetic field, most probably leads to a gross underestimate of κ. Belcher's results for Alfven waves also indicate that this slab model is incorrect.

The consensus was that a major effort should be expended in the fabrication of a more correct model for **P** (k). This process will involve examination of the magnetic records of two or more spacecraft operating simultaneously within one magnetic correlation length of one another. A realistic model will properly include in **P** (k) the effect of tangential discontinuities so that no further effort to remove them in an ad hoc fashion is necessary.

Concurrently, efforts to explore the sensitivity of κ to choices of **P** (k) should be continued. These choices might be based solely on plausibility or might follow, for example, from a complementary calculation of wave generation and propagation in the expanding solar wind.

The opportunity for valuable and significant work in many areas lies ahead.

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EPILOGUE

After listening for hours to Workshop tapes and discussing with each other implications of the discussion on them, we concur strongly with certain conclusions. We feel that theory is definitely on the right track in pursuing fundamentally a momentum-space statistical description of the interaction between cosmic rays and the magnetic fluctuations. We have also become increasingly hopeful that theoretical techniques developed in this area, where the cosmic rays are "test particles" which do not affect the magnetic fields, may be useful in the full-blown plasma turbulence problem, where Maxwell's equations are a constraint relating the fields to the particles. Nonlinear theories are clearly necessary in certain regions of phase space. To distinguish the merits and limitations of different nonlinear theories, computer simulation experiments are a further necessity. Simulations which reflect the full three-dimensional nature of the interplanetary medium need to be developed. We believe it desirable that computer experiments be carried out by more than one group; the attendant competition and independence of approach can lead to more rapid progress. Finally, our ultimate contact with and verification of reality depends on a far more detailed knowledge of the interplanetary magnetic field fluctuations than now exists. In particular, theory needs as input the tensor power spectrum $\mathbf{P}(\mathbf{k})$ as a function of wave vector $\mathbf{k}$. To obtain $\mathbf{P}$, simultaneous measurements from two or more satellites located within a correlation length or so are required. Since the cosmic rays typically spend ~50-100 days in the solar system (O'Gallagher, 1973), the measurements should be made systematically over such intervals. It also would be valuable to obtain power spectrum measurements at high interplanetary latitudes, since modulation may not be confined to the ecliptic plane.

REFERENCES

Burlaga, L. F., 1971, *J. Geophys. Res.* **76**, 4360.

Dupree, T. H., 1966, *Phys. Fluids* **9**, 1773.

Earl, J. A., 1974, *Astrophys. J.* **193**, 231.

Fisk, L. A., B. Kozlovsky, and R. Ramaty, 1974, *Astrophys. J.* **190**, L35.

Fisk, L. A. and J. W. Sari, 1973, *J. Geophys. Res.* **78**, 6729.

Jokipii, J. R., 1966, *Astrophys. J.* **146**, 480.

Jokipii, J. R., and P. J. Coleman, Jr., 1968, *J. Geophys. Res.* **73**, 5495.

Jones, F. C., T. B. Kaiser, and T. J. Birmingham, 1973, *Phys. Rev. Letters* **31**, 485.

Kaiser, T. B., 1973, Dissertation, Univ. of Maryland.

Lupton, J. E., and E. C. Stone, 1973, *J. Geophys. Res.* **78**, 1007.

McCracken, K. G., V. R. Rao, and R. P. Bukata, 1967, *J. Geophys. Res.* **72**, 4293.

McDonald, F. B., B. J. Teegarden, J. H. Trainor, and W. R. Webber, 1974, *B.A.P.S.* **19**, 433.

O'Gallagher, J. J., 1973, *Papers, 13th International Cosmic Ray Conference*, 1135.

Owens, A. J., 1974a, *J. Geophys. Res.* **79**, 895.

Owens, A. J., 1974b, *Astrophys. J.* **191**, 235.

Owens, A. J., and J. R. Jokipii, 1972, *J. Geophys. Res.* **77**, 6639.

Quenby, J. J., G. E. Morfill, and A. C. Durney, 1974, *J. Geophys. Res.* **79**, 9.

Roelof, E. C., 1966, Dissertation, Univ. of California, Berkeley.

Roelof, E. C., 1968, *Can. J. Phys.* **46**, S990.

Solodyna, C. V., and J. W. Belcher, 1974, *EOS* **55**, 414.

Teegarden, B. J., F. B. McDonald, J. H. Trainor, E. C. Roelof, and W. R. Webber, 1973, *Astrophys. J.* **185**, L155.

Völk, H. J., 1973, *Astrophys. Space Sci.* **25**, 471.

Weinstock, J., 1969, *Phys. Fluids* **12**, 1045.

NASA-Langley, 1975